Auctions on Advertisement

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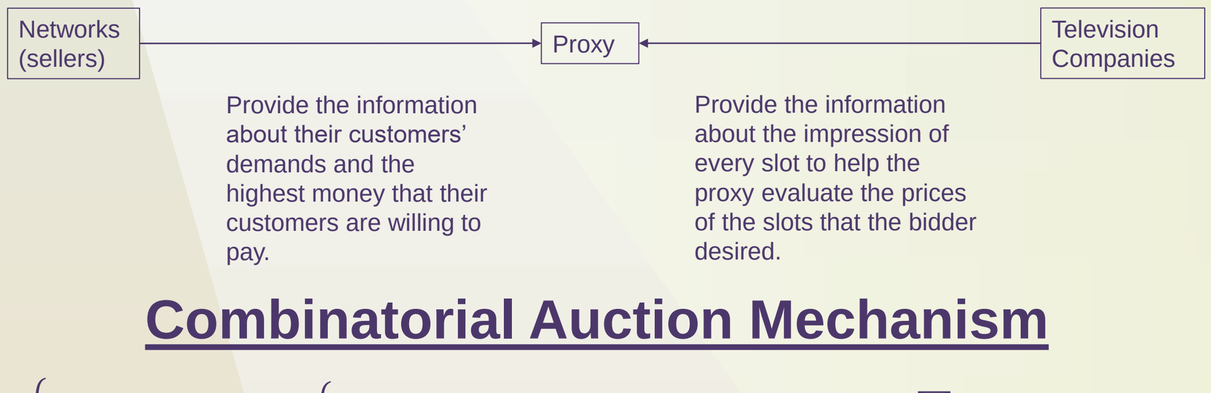
Abstract

In this project, we study a combinatorial proxy auction process in which advertising agencies place orders with the networks for their desired commercial slots to expose their products to targeted audience. Based on a widely used model for online auction of airline tickets formulated by Marta Eso in 2002, we build an integer programing model to demonstrate the advertisement auction mechanism behind the scene. Furthermore, to establish an efficient way of allocating the bids, we apply two methods (impression-based and demographic-based approaches) to evaluate the bidders' orders, and therefore one can examine several recent algorithms for winner determination problem.

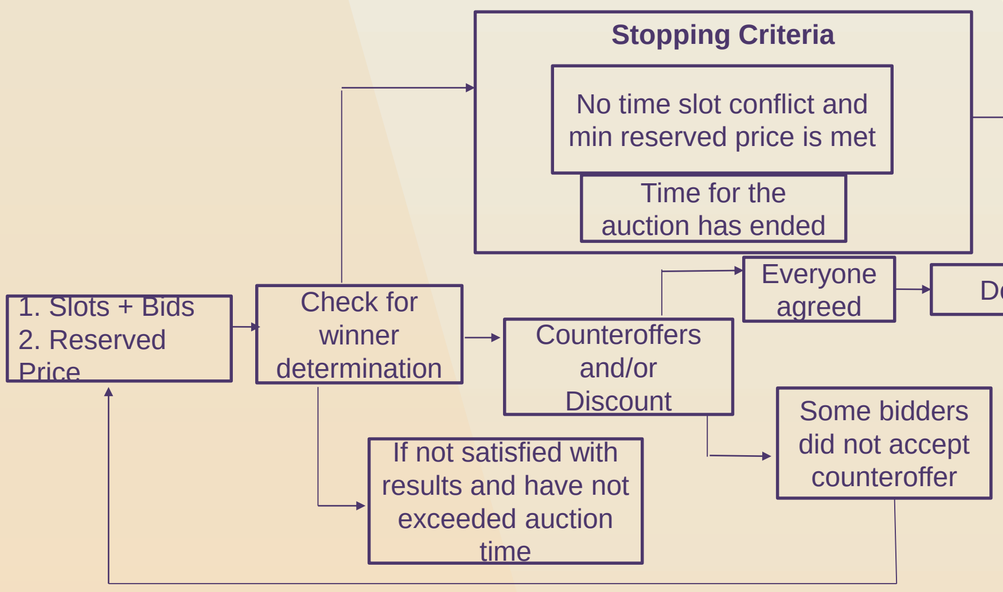
Background



Combinatorial Auction Mechanism

$$\begin{cases} Sealed-Bid\ Auction \begin{cases} First-Price\ Principle\ R^{*1st} = \sum_{i\in w} p_i^{*1st} \\ Vickrey-Clarke-Groves\ (VCG) Mechanism\ R^{*VCG} = \sum_{i\in w}[\alpha_i - \sum_{k\neq i} b_k(S)] \end{cases} \\ Dynamic\ Auction \begin{cases} \text{Ascending } Proxy\ Auction \\ Clock-Proxy\ Auction \\ Combinatorial\ Clock\ Proxy \end{cases} \end{cases}$$

Methods



Mathematical Approach to Algorithm Engineering

1. Determine a P-Value Matrix for the value of each slot before bidders are allowed to bid.
2. Remove the bids that are lower than the reserved price in the P-Matrix
3. Bidders with the highest bidding price for each time slot section will win the bid with the constraint: the number of chosen bids can not exceed the total number of placements for one slot. If there is a tie or over-valued relationship, one will use the technique in auction mechanism (e.g., Clock-Proxy Auction)
4. Remove all accepted bids and consider discounts and counter offers by developing a possible package of slots for the bidder using their highest placed bids.

Integer Programming Formulation in a Single Round

A slot is a designated time on a particular network (e.g., ABC on October 1st, 2016, from 10:11–10:14 am) which can run one or more commercials in pods. Here, we use S to denote the set of the slots:

$S = \{1,2,3,\dots,m\}$

$\pi_j, j = 1,2,3,\dots,m$ to denote the number of pods in slot j.

$r_j, j = 1,2,3,\dots,m$ to denote the reserved price for each slot j.

x_i to denote whether the bid is accepted or not.

$$x_i = \begin{cases} 1, if\ the\ bid\ is\ accepted \\ 0, if\ not \end{cases}, i = 1,2,3,\dots,n$$

b_{ij} to represent whether bidder i wants the slot j.

$$b_{ij} = \begin{cases} 1, if\ the\ bidder\ i\ bid\ on\ the\ slot\ j \\ 0, if\ not \end{cases}, i = 1,2,3,\dots,n; j = 1,2,3,\dots,m$$

We consider the following Integer Programming formulation:

$$max\left\{\sum_{i=1}^{n}[\sum_{j=1}^{m}(b_{ij}p_{ij} - b_{ij}r_j)]\,x_i\right\}$$

such that

$$\begin{cases} \sum_{i=1}^{n} b_{ij}x_i \le \pi_j, i = 1,2,3,\dots,n;\ j = 1,2,3,\dots,m \\ p_{ij} \ge r_j, i = 1,2,3,\dots,n;\ j = 1,2,3,\dots,m \\ x_i = \{0,1\} \end{cases}$$

Evaluation of Slots' Values

To evaluate the values of the slots, we have two different evaluating methods:

It is more statistically convenient to gather the information of impressions based on the age range and the gender value.

Method 1: Impression-Based Approach

$$p_{ij} = (total\ bid\ i)(impressions\ for\ slot\ j)\left(\frac{value\ to\ i}{desired\ impression}\right)^{-1} = u_i(\sum_{a,g} v_{jag})w_i$$

$$w_i = (\sum_{j,a,g} b_{ij}v_{jag})$$

$$p^*{}_{ij} = b_{ij}p_{ij}$$

$$\sum_j p^*{}_{ij} = u_i, \qquad i = 1,\dots,i_{max}$$

p_{ij} = value to order i of slot j

u_i = ensures that a buyer is only awarded a slot they initially desired

$b_{ij} = slots\ desired$

Method 2: Demographic-Based Approach

$$p_{ij} = (total\ bid\ i)\sum_{a,g}\left[\begin{matrix} impressions\ for\ slot\ j\ and \\ demographic\ (a,g) \end{matrix}\right]\left[\frac{value\ to\ i}{desired\ impression\ in\ (a,g)}\right] = u_i\sum_{a,g} v_{jag}w_{iag}$$

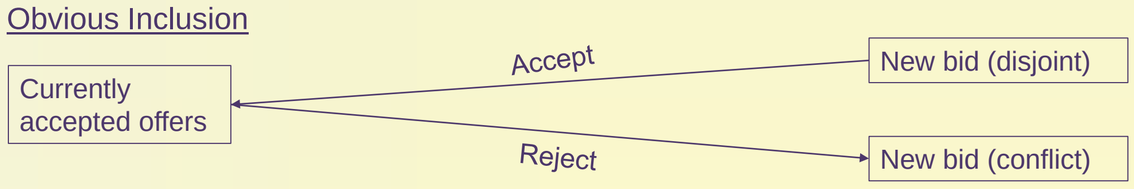
$$u_i = \sum_j b_{ij}p_{ij} = u_i\alpha_i\sum_j b_{ij}[\sum_{a,g} v_{jag}(\sum_j b_{ij}v_{jag})]$$

$$\frac{1}{\alpha_i} = \sum_{a,g}(\sum_j b_{ij}v_{jag})^2$$

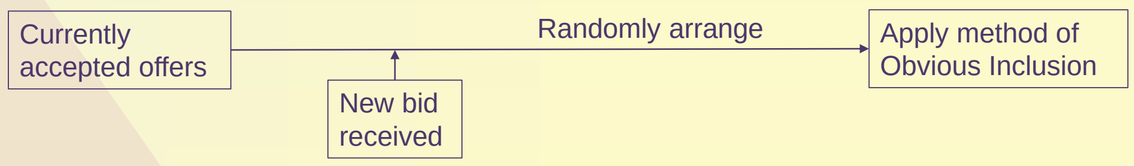
$$w_{iag} = \sum_j b_{ij}v_{jag}[\sum_{a,g}(\sum_j b_{ij}v_{jag})^2]^{-1}$$

Computation Algorithm of WDP

Obvious Inclusion



Simple Greedy Heuristic



IP Based Heuristic

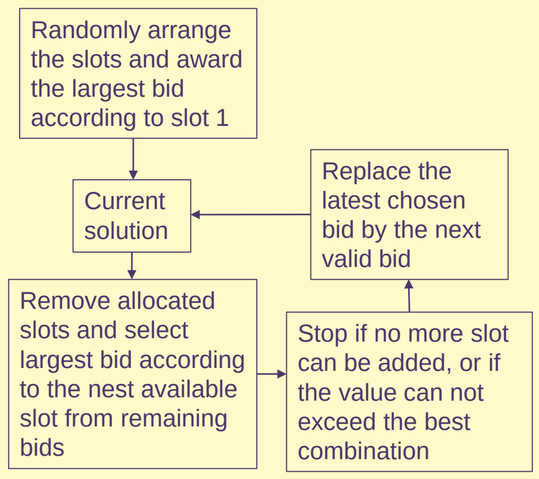
Bid / Slot	1	2	3	4
A	0	340	1000	0
B	0	500	900	810
C	1000	1100	0	0
D	450	0	0	830
E	0	620	340	120

Results:

Start from Slot A: 1000+900+340+1000+450=3690

Start from Slot C: 1100+500+340+620=2560

Algorithm



Conclusion

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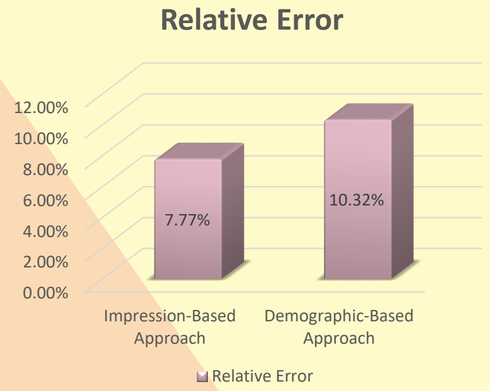
By comparison of these two approaches, the demographic-based approach is proved to be more sensitive to the impressions of the demographic since a normally distributive perturbation with standard deviation 5% of the original value is added to the prices of the orders and the relative errors for the impression-based approach and demographic-based approach are 7.77% and 10.32%, respectively.

As the results show, the demographic-based approach could represent the real value more accurately.



Reference

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